Programming of the “Banks” Model: Mathematically Modeling Wind and Rain’s Effect on Surface Reaeration

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ABSTRACT
Dissolved oxygen is essential to the respiratory metabolism of most aquatic organisms. Its dynamics and distribution within inland waters is governed by a balance between inputs from the atmosphere and photosynthesis and losses from chemical and biotic oxidation (Wetzel, 2001). Commonly discussed factors affecting distribution of dissolved oxygen include diffusion, wind driven mixing, and biochemical oxygen demand, but rarely, if ever, mention input from rain. With rain’s dissolved oxygen content nearly saturated below 15°C (Richards, 1917), it becomes of considerable interest to investigate the input of dissolved oxygen into a water body through rain. To better understand the impact of rain on the dissolved oxygen content of a water body, the theories provided by the literature of R.B. Banks are used to develop a mathematical model with the sole purpose of estimating oxygen input due to rain mass input and/or wind speed.

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HOW DOES WIND AND RAIN AFFECT D.O. CONTENT? There are three mechanisms through which rain and wind transfer oxygen in to a water body and, thus, influence its reaeration; 1- the effect of wind driven mixing; 2- the turbulence induced by the falling rain drops; and 3- the direct addition of dissolved oxygen from each saturated rain drop.

THE EFFECT OF WIND & RAIN INDUCED TURBULENCE. According to the literature of R.B. Banks, change in a water body’s dissolved oxygen (DO) content due to wind driven mixing and rain induced turbulence can be characterized through use of a coefficient known as the total surface reaeration coefficient, $K$.

This coefficient, with units of $s^{-1}$, is determined from two other coefficients; 1- the reaeration coefficient of wind, $K_w$, and 2- the reaeration coefficient of rain induced turbulence, $K_r$. This is expressed mathematically by Eqn. 1 below in which $K_w$ is an empirically found constant equal to 0.246 $cm^2/s$ and $H$ is the mixed layer depth of the respective water body.

$$K_2 = \frac{K_o - \left(K_wK_r \right) + K_r}{H}$$  \hspace{1cm} (1)

It might become questionable after first reading the above equation as to how it relates directly to any meteorological variables. The answer to this question lays in the solving of the other coefficients ($K_o$ and $K_w$). It is in the solving of these coefficients that input of measured meteorological variables (wind and rain rate) is done.

The reaeration coefficient of wind, $K_w$, is solved for using Eqn. 2 and wind speed, $U$, in meters per second.

$$K_w = 10^{-6} \left(8.43 \frac{U^2}{2} - 3.67U + 0.43U^2\right)$$  \hspace{1cm} (2)

The reaeration coefficient of rain induced turbulence, $K_r$, having been found by Banks to be directly proportional to the power input of rainfall, requires that power input of rain fall be determined. Using Eqn. 3 below, power input of rain fall, $P_r$, is determined in units of $dyne \cdot cm^{-2} \cdot s^{-1}$ through use of elevation, $z$, in meters and rain rate, $r$, in millimeters per hour.

$$P_r = (2.39 + 0.103 * 10^{-3}z)r^{1.26}$$  \hspace{1cm} (3)

Having determined the power input of rain, the reaeration coefficient of rain induced turbulence, $K_r$, can be obtained through use of Eqn. 4.

$$K_r = (0.0283 * 10^{-3} cm^2/dyne^{-1}) P_r$$  \hspace{1cm} (4)

With the reaeration coefficient of wind, $K_o$, and the reaeration coefficient of rain induced turbulence, $K_r$, determined, the total surface reaeration coefficient, $K_2$, can be found through use of Eqn. 1 with input of the respective water bodies mixed layer depth, $H$.

CHANGE IN D.O. DUE TO WIND-DRIVEN MIXING AND RAIN INDUCED TURBULENCE. A mass balance on an elemental level yields Eqn. 5 for the horizontal ($x$, $y$) distribution of dissolved oxygen in a vertically well-mixed lake or lagoon where $L$ = concentration of BOD, $C$ = concentration of DO, $t$ = time, $K_i$ = deoxygenation coefficient, $K_2$ = reaeration coefficient, $E_x$ & $E_y$ = turbulent diffusion coefficient in the x and y direction respectively, and $C^*$ = sources and sinks of DO.

$$\frac{\partial C}{\partial t} + \nabla \cdot \nabla \cdot \left(\frac{\partial C}{\partial x} \nabla + K_2 \nabla C \right) = \nabla \cdot \nabla \left(\frac{\partial C}{\partial x} \nabla \right) + \frac{\partial}{\partial y} \left(\frac{\partial C}{\partial y} \nabla \right) + C^*$$  \hspace{1cm} (5)

In such a condition that there is a well mixed and homogenous body of water with no biochemical oxygen demand, Eqn. 5 can be simplified to Eqn. 6 and, thus, the
change in DO content as a result of wind-drivin mixing and rain induced turbulence is solved.

\[ \frac{dc}{dt} = K_2C \]  

(6)

With the purpose of modeling not only being the characterization of past phenomena, but also the future, it becomes of considerable interest to project future changes in DO content. Integrating Eqn. 6, Eqn. 7 is obtained with \( C_0 \) being the initial water body DO concentration and \( t \) equal to time since initial DO measurement.

\[ C = e^{K_2t} \ln C_0 \]  

(7)

D.O. CONTENT OF RAINDROPS. Having discussed the first and second mechanisms through which oxygen is transferred and reaeration occurs, there is but one left to be discussed… direct addition from oxygen saturated rain. The dissolved oxygen content of a rain drop is said to be at saturation as it falls through the atmosphere and was reported as such by Eric Richards in 1914 for observations up to 15°C. This allows for the dissolved oxygen content of a rain drop, \( C_r \), in mg · L⁻¹ to be approximated using an equation that solves for \( O_2 \) saturation of deionized water as a function of temperature in degrees Celsius, and atmospheric pressure and vapor pressure in Torrs (Eqn. 8).

\[ C_r = \frac{(P - p)\cdot 0.678}{35 + T_r} \]  

(8)

The temperature of a rain drop, \( T_r \), was reported by Anderson et al (1998) & Flament et al in (1995) to be that of the wet-bulb temperature. Thus, the wet-bulb temperature must be determined for use of Eqn. 8. This, however, presents a problem. There is rarely data available for direct measurements of wet-bulb temperature. This creates the need to determine the wet-bulb temperature through use of a popular measurement, dew-point, \( T_d \), in this case. Through input of the dew-point temperature into Eqn. 9, the wet-bulb temperature, \( T_w \), is determined.

\[ T_w = [T_{air} - 9.8 \cdot 0.125 \cdot (T_{air} - T_d)] + \ldots \]

\[ + (9.8 \cdot 0.125 \cdot (T_{air} - T_d)) \]  

(9)

Having solved for the wet-bulb temperature, atmospheric pressure, \( P \), is used in Eqn. 8 along with water vapor pressure, \( p \), determined using the Clausius-Clapeyron equation (note: pressure is kept in units of torrs). Using measured values of \( P \), \( T_{air} \) (air temperature), and \( T_d \), along with the calculated values of \( p \) and \( T_w \), the dissolved oxygen content of rain drops in a rain fall (\( C_r \)) are approximated.

DIRECT INPUT OF D.O. FROM RAIN. The direct input of dissolved oxygen from an oxygen saturated rainfall is far more simplistic mathematically than determining the oxygen transferred by wind-drivin mixing and rain induced turbulence. The dissolved oxygen transferred directly to a water body through rain fall is simply solved for using Eqn. 10. This equation, provided by Banks in his 1984 paper, determines the amount of dissolved oxygen transferred at any given moment, \( M \), through input of rain’s dissolved oxygen content, \( C_r \), in mg · L⁻¹, rain rate, \( r \), in cm · hr⁻¹, and mixed layer depth in cm, \( H \).

\[ M = \frac{C_r \cdot r}{H} \]  

(10)

SCHMIDT DATA ANALYSIS. Analysis of Schmidt’s 2010 experimental data is done in the Banks model with a constant wind speed (\( U \)) and rain power input (\( P_r \)) of zero. This is due to Schmidt’s experiments having been performed indoors where the atmosphere is relatively static and due to the oxygen saturated water (simulated raindrops) being added to the water body from a height on the order of a couple inches. Dropping the saturated water from such a small height yields a comprehensive power input of less than 10⁶, thus making power input negligible.

With the limitations of Schmidt’s experimental data mentioned above, any analysis must be purely based on a mass balance of water and oxygen that does not consider rain induced turbulence or wind-drivin mixing. Thus, Eqn. 10 is used.

Because Schmidt had not directly measured the depth of the water bodies in his experiments, their depth had to be approximated using initial and final water body masses and container dimensions. Further complicating Schmidt’s data was the change in water body depth, \( H \), with time as oxygen saturated water was added. \( H \) is therefore determined by Eqn. 11, thus allowing for the total oxygen transferred, \( M_T \), to be determined.

\[ H = H_0 + r \cdot t \]  

(11)

In the case of Schmidt’s experiments, the initial depth, \( H_0 \), was 10.84 cm & 11.84 cm for the first and second data sets respectively. The rain rate for the first data set was 6.11 mm · hr⁻¹ and 42.9 mm · hr⁻¹ for the second dataset. A graph of the modeled change in depth for both dataset 1 and 2 of Schmidt’s 2010 experiments is shown by Figures 1 and 2.

Using the modeled water body depths determined from Eqn. 11, instantaneous oxygen transfer, \( M \), is determined using Eqn. 10. The modeled oxygen transfer determined for the first and second datasets of Schmidt’s experiments are shown in Figures 3 and 4 respectively.

It is clearly shown that as the depth of the water body, \( H \), increases (shown in figures 1 & 2) that the oxygen transferred to the water body, \( M \), can be expected, theoretically speaking, to decreases (shown in figures 3 & 4).

Summing of all time step values of \( M \), the total oxygen transferred, \( M_T \), is determined. Values for the \( M_T \) of both of Schmidt’s datasets are shown below in Table 1. The modeled dissolved oxygen content of the water bodies in the first and second datasets are shown in Figures 5 & 6 respectively.

| TABLE 1. Modeled Total Oxygen Transferred, \( M_T \), for both Schmidt Datasets. |
|---|---|
| Dataset # | \( M_T \) (mg L⁻¹) |
| 1 | 0.25 |
| 2 | 1.51 |
FIGURE 1. Modeled change of water body depth ($H$) with time ($t$) for the first dataset of Schmidt’s 2010 experiments. Note: Time is from start of oxygen saturated water addition at $t = 3580$ to $t = 5380$ (Total time = 15 min.)

FIGURE 2. Modeled change of water body depth ($H$) with time ($t$) for the second dataset of Schmidt’s 2010 experiments. Note: Time is from start of oxygen saturated water addition at $t = 7260$ to $t = 9060$ (Total time = 15 min.)

FIGURE 3. Modeled change in water body oxygen concentration $C_w$ with time ($t$) for the first dataset of Schmidt’s 2010 experiments. Note: Time is from start of oxygen saturated water addition at $t = 3580$ to $t = 5380$ (Total time = 15 min.)

FIGURE 4. Modeled change in water body oxygen concentration $C_w$ with time ($t$) for the second dataset of Schmidt’s 2010 experiments. Note: Time is from start of oxygen saturated water addition at $t = 7260$ to $t = 9060$ (Total time = 15 min.)

FIGURE 5. Modeled water body dissolved oxygen content for Schmidt’s first 2010 experimental dataset.

FIGURE 6. Modeled water body dissolved oxygen content for Schmidt’s second 2010 experimental dataset.
Having modeled the expected change in dissolved oxygen content of the two water bodies in Schmidt’s 2010 experiments, the modeled “theoretical” change can be compared to the actual measured values. A comparison of the actual and modeled dissolved oxygen concentrations for the water bodies in both datasets are shown in Figures 7 & 8.

**CONCLUSION OF SCHMIDT DATA ANALYSIS.** The differences between modeled and actual dissolved oxygen content shown in figures 7 & 8 suggest that application of the Banks model to the conditions of Schmidt’s 2010 experiments is inappropriate. The model clearly shows, based on a mass balance, that the dissolved oxygen content of the water bodies in Schmidt’s experiments should have increased significantly more than was shown through direct measurement.

Two potential causes are suggested for this occurrence; 1- the rain rate approximated using a start to end massing difference inaccurately modeled the rain rate as being faster than actual, 2- exposure of the water bodies to the atmosphere days before each experiment resulted in a change in their salinity and/or pH, thus limiting the total amount of dissolved oxygen they could contain at saturation.

**WLIS DATA ANALYSIS.** Using rain rate data from Republic, Westchester, and LaGuardia Airports and meteorological data from the Western Long Island Sound (WLIS) Buoy 44040, instantaneous values of $K_2$ and $M$ were determined for the time period of 198th to 217th Julian day in the year 2004. With no mixed layer depth data available, division by $H$ in Eqns. 1 & 10 was omitted. As a result $K_2$ and $M$ values were kept in units of cm·s$^{-1}$ and cm·mg·L$^{-1}$·s$^{-1}$ respectively and total oxygen transferred from direct rain input, $M_T$, was kept in units of cm·mg·L$^{-1}$. Please note that missing data points in meteorological and rain rate data were interpolated linearly. A map showing the location of the airports and buoy is shown in Figure 9.

Input of WLIS buoy wind speed data into Eqn. 2 gave the reaeration coefficient of wind, $K_0$ (Figure 10). An average rain rate (Figure 11) determined from averaging the data of the three previously specified airports was used in Eqns. 3 & 4 to obtain the reaeration coefficient of rain induced turbulence, $K_r$ (Figure 13). Determined $P_T$ and $K_r$ values are depicted in Figures 12 & 13. Resulting $K_2$ values are shown in Figure 14.

Equations 8, 9, and 10 were used to determine the amount of dissolved oxygen added to WLIS as a result of rain’s dissolved oxygen content. The resulting values of $M$ are shown in Figure 15. Summation of all $M$ values gives total oxygen transferred from direct rain input, $M_T$, of 0.0235 cm·mg·L$^{-1}$.

| TABLE 2. Statistics of $K_0$, $P_T$, $K_r$, $K_2$, and $M$ for WLIS |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $K_0$ (cm·s$^{-1}$) | 5.3 x 10$^4$ | 0.0017 |
| $P_T$ (dyne cm$^{-2}$·s$^{-1}$) | 6.7 | 56.32 | 307 (dyne cm$^{-2}$) |
| $K_r$ (cm·s$^{-1}$) | 1.9 x 10$^4$ | 0.0016 |
| $K_2'$ (cm·s$^{-1}$) | 5.221 x 10$^4$ | 0.0017 |
| $M$ (cm·mg·L$^{-1}$) | 5.1 x 10$^3$ | 0.0031 | 0.024 (cm·mg·L$^{-1}$) |

**FIGURE 7.** Modeled and actual dissolved oxygen content for Schmidt’s first 2010 experimental dataset.

**FIGURE 8.** Modeled and actual dissolved oxygen content for Schmidt’s second experimental dataset.

**FIGURE 9.** Location of sites for data analysis. Green triangles are representative of land-based stations. Red diamonds represent buoy locations.
FIGURE 10. Surface reaeration coefficient of wind, $K_0$, determined from WLIS wind speed buoy data through use of Eqn. 2 for the time from of 198th to 217th Julian day.

FIGURE 11. Averaged rain rate for WLIS from the 198th Julian day to the 217th Julian day for the year of 2004.

FIGURE 12. Rain power input, $P_T$, calculated by the Banks model from the 198th Julian day to the 217th Julian day 2004 buoy and meteorological data.

FIGURE 13. Surface reaeration coefficient of rain induced turbulence, $K_r$, calculated by the Banks model from the 198th Julian day to the 217th Julian day 2004 buoy and meteorological data.

FIGURE 14. Total surface reaeration coefficient, $K_2$, before division by the mixed layer depth, $H$, as calculated by Banks model from 198th Julian day to the 217th Julian day 2004 buoy and meteorological data.

FIGURE 15. Oxygen transferred, $M$, directly from addition of rainfall’s dissolved oxygen content, $C_r$, as calculated by Banks model from 198th Julian day to the 217th Julian day 2004 buoy and meteorological data.
CONCLUSION OF WLIS DATA ANALYSIS. The Banks Model applied to WLIS for the 198th to 217th Julian day time frame of 2004 indicates that on average the total surface reaeration coefficient, $K_2$, was $5.52 \times 10^{-4}$ $\text{cm} \cdot \text{s}^{-1}$ and obtained a maximum value of 0.0017 $\text{cm} \cdot \text{s}^{-1}$. Total power input from rain averaged 0.67 $\text{dyne} \cdot \text{cm}^{-2} \cdot \text{s}^{-1}$ and had a maximum of 307 $\text{dyne} \cdot \text{cm}^{-2} \cdot \text{s}^{-1}$. The reaeration coefficient of rain induced turbulence, $K_r$, had an average of $1.9 \times 10^{-5}$ $\text{cm} \cdot \text{s}^{-1}$ and a maximum of 0.0016 $\text{cm} \cdot \text{s}^{-1}$. Direct addition of dissolved oxygen from saturated raindrops was on average $5.1 \times 10^{-5}$ $\text{mg} \cdot \text{L}^{-1} \cdot \text{s}^{-1}$ with a maximum of 0.0031 $\text{mg} \cdot \text{L}^{-1} \cdot \text{s}^{-1}$ and a total transport of 0.024 $\text{mg} \cdot \text{L}^{-1}$. 

Comparison of the reaeration coefficients in Table 2 suggest that rain induced turbulence had a reaeration coefficient one order of magnitude less than that of wind on average. Despite this observation though, the reaeration coefficient of rain induced turbulence had a maximum almost identical to the maximum of wind’s reaeration coefficient. This suggests that rain induced turbulence, although less influential on average, did have the ability to be just as effective as wind in the reaeration of WLIS.

Considering the proportionality of $K_r$ to the power input of rain (refer to Eqn. 4), it can be suggested that the heaviest of rain falls that occurred were comparable in effect (in terms of generated turbulence) to that of the strongest occurring winds. With rain induced turbulence alone being just as effective as wind, it can be suggested that heavy rains can be more effective once the direct addition of their dissolved oxygen content is considered. Figure 11 (the average rain rate for WLIS) indicates that there were a minimum of at least four rain events over WLIS during the time period analyzed. When compared to Figure 14 it is evident that the total surface reaeration coefficient increased with every rain event.

FUTURE WORK. Due to the lack of available dissolved oxygen and mixed layer depth data for WLIS, the Banks Model wasn’t able to become fully functional in its ability to predict actual DO content changes in WLIS. To better validate the findings of this research it would be very desirable to obtain mixed layer depth and dissolved oxygen data for WLIS. A possible solution to the lack of mixed layer depth data would be to model the mixed layer’s depth through use of the PWP model.

Because the WLIS buoy is still fairly new and one of its first datasets was used in this analysis, it is thought that future data sets will prove more beneficial in the Banks Model as more detailed measurements become available.

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REFERENCES.


MATLAB CODE FOR LABORATORY ANALYSIS

close all;
clear all;

Case = 1
Test = 0;
Trial1 = 1
Trial2 = 0

Banks = 1; % Banks = 1 to run Banks Model

% TTTTTTTTTTT 1111
% T   1111
% T   1
% T   1
% T   11111
%******************************************************************************
%LOAD MTR-499 Water Body DO Sensor Voltage Data
[rRun1Time rRun2Time rRun3Time rRun3TimeS2 rVol1 rVol2 rVol3 rVol3_s2 rVol3_s1_2] = textread('C:\Senior Research\MTR 499\MTR 499 Data\Analysis Data\Oxygen - Graph 1\AllOxVol.txt','%u %u %u %f %f %f %f %f','%f','headerlines',1);

% Case selection
% 0 1 2 3 4 5
CaseVol_s = [1 1800];
CaseVol_f = [length(rRun1Time) 3601];
NsVol = CaseVol_s(Case+1);
NfVol = CaseVol_f(Case+1);
NtVol = NfVol-NsVol+1;

disp('Trial 1 Water Body Oxygen Concentration Data, selected')

%-----------------------------------------------------------------------------
% Define the variables
Run1Time = rRun1Time(NsVol:NfVol);
Run2Time = rRun2Time(NsVol:NfVol);
Run3Time = rRun3Time(NsVol:NfVol);
Vol1 = rVol1(NsVol:NfVol);
Vol2 = rVol2(NsVol:NfVol);
Vol3 = rVol3(NsVol:NfVol);
Vol3_s2 = rVol3_s2(NsVol:NfVol);
Vol3_s1_2 = rVol3_s1_2(NsVol:NfVol);

%LOAD MTR-499 Water Body Oxygen Concentration Data
[rRun1Time rOx1 rRun2Time rOx2] = textread('C:\Senior Research\MTR 499\MTR 499 Data\Analysis Data\Oxygen - Graph 1\WB_OxConChB_All.txt','%u %f %u %f','headerlines',1);

% Case selection
% 0 1 2 3 4 5
CaseOxy_s = [1 1800];
CaseOxy_f = [length(rRun1Time) 3601];
NsOxy = CaseOxy_s(Case+1);
NfOxy = CaseOxy_f(Case+1);
NtOxy = NfOxy-NsOxy+1;

disp('Trial 1 Water Body Oxygen Concentration Data, selected')
% Define the variables
Run1Time = rRun1Time(NsOxy:NfOxy);
Run2Time = rRun2Time(NsOxy:NfOxy);

Ox1 = rOx1(NsOxy:NfOxy);
Ox2 = rOx2(NsOxy:NfOxy);

%LOAD MTR-499 Water Body Temperature Data
[rRun1Time rRun2Time rRun3Time rTemp1 rTemp2 rTemp3] = textread('C:\Senior Research\MTR 499\MTR 499 Data\Analysis Data\WB Temp - Graph 2\WB_TempChA_All.txt','%u %u %u %f %f %f','headerlines',1);

% Case selection
%              0                                1     2      3   4    5
CaseTemp_s = [1                    1800];
CaseTemp_f = [length(rRun1Time)    3601];
NsTemp = CaseTemp_s(Case+1);
NfTemp = CaseTemp_f(Case+1);
NtTemp = NfTemp-NsTemp+1;

disp('Trial 1 Water Body Temperature, selected')

% Define the variables
Run1Time = rRun1Time(NsTemp:NfTemp);
Run2Time = rRun2Time(NsTemp:NfTemp);
Run3Time = rRun3Time(NsTemp:NfTemp);

Temp1 = rTemp1(NsTemp:NfTemp);
Temp2 = rTemp2(NsTemp:NfTemp);
Temp3 = rTemp3(NsTemp:NfTemp);

% Setting up Reference line for when rain addition began.
liney = [6 7.5];
linex = [3580 3580];

figure (101)
%subplot(2,1,1)
hold on;
stairs(Run1Time,Ox1,'b-')
stairs(Run1Time,Ox2,'r-')
%stairs(Run1Time,Ox3,'g-')
plot(linex, liney, 'k--')
hold off;
box on;
%axis([1780 5380 3.0 3.5])
legend('Trial 1', 'Trial 2', 0)
ylabel('Oxygen Concentration (mg L^{-1})')
xlabel('Time (seconds)')
figure(102)
%subplot(2,1,2)
hold on;
stairs(Run1Time,Temp1,'b-')
stairs(Run1Time,Temp2,'r-')
%stairs(Run1Time,Ox3,'g-')
%plot(linex, liney, 'k--')
hold off;
box on;
%axis([1780 5380 19.2 19.7])
legend('Trial 1', 'Trial 2', 0)
ylabel('Temperature ( C\text{o} )')
xlabel('Time (seconds)')

if Banks == 1:
    if Test == 1:
        C_o = 7.3; % Oxygen Concentration just prior to rain's addition
        C_r = 9.21; % mg/l - pressure 765 torr @ 19.5 C
        U = 11.1; % Wind Magnitude (m/s)
        r = 15; % mm/hr experimental with water density @ 20 C
        K_star = 0.0246; % b1/b2 (cm/s)
        z = 2000; % Altitude above MSL in meters
        C_s = C_r; % Oxygen Saturation level of Water Body
    end;
    if Trial1== 1:
        C_o = 7.3; % Oxygen Concentration just prior to rain's addition
        C_r = 8.94; % mg/l - pressure 765 torr @ 19.5 C
        U = 0; % Wind Magnitude (cm/s)
        r = 6.11; % mm/hr experimental with water density @ 20 C
        K_star = 0.0246; % b1/b2 (cm/s)
        z = 0; % Altitude above MSL in meters
        C_s = C_r; % Oxygen Saturation level of Water Body
        H_o = 10.84; % depth in centimeters
    end;
    if Trial2 == 1:
        C_o = 6.5; % Oxygen Concentration just prior to rain's addition
        C_r = 9.06; % mg/l - pressure 765 torr @ 19.5 C
        U = 0; % Wind Magnitude (cm/s)
        r = 42.9; % mm/hr experimental with water density @ 20 C
        K_star = 0.0246; % b1/b2 (cm/s)
        z = 0; % Altitude above MSL in meters
        C_s = C_r; % Oxygen Saturation level of Water Body
        H_o = 11.84; % depth in centimeters
    end;
end;

i=1;
for i = i:length(Run1Time);
    K_o(i) = (10^(-6))*((8.43*(U^(1/2)))-(3.67*U)+(0.43*(U^2)))*100; % Determining Oxygen Transfer Coefficient due to only wind
    P_t(i) = (2.39 + (0.103*(10^(-3))*z))*r^(1.26); % Determining the Total Power input due to rain
    K_r(i) = (0.0283 * (10^-3)) .* P_t(i); % Determining the Oxygen Transfer Coefficient due to rain
    K_t_NT(i) = (K_o(i)) - ((K_o(i).*K_r(i))./(K_star)) + K_r(i); % Determining the total oxygen transfer coefficient
    K_2 = K_t_NT/H_o;
end;
K_t = K_t_NT'; % Transposing K_t Matrix for comparison with real data.
disp('K_t Determined Successfully...')

% Determining new Depth (H) with time
h=1;
for h = h:length(Run1Time-1);
    H(h) = ((r/36000)*h)+H_o;
end;

disp('Change in Depth Determined Successfully...')

% Projected Change in DO with no Sinks
Cc(1) = C_o;
for k = k:length(Run1Time-1);
    Cc(k) = Cc(k-1) + K_2(k-1)*Cc(k-1);
end;

disp('Projected Change with no Sinks Determined Successfully...')

%M_total Amount of Oxygen Transferred
M(1) = (((r * C_r)/36000))/H(1); % + (K_r(1)*(C_s-C_o));
M_t(1) = C_o + M(1);
for j = j:length(Run1Time-1);
    M(j) = (((r * C_r)/36000))/(H(j)); % + (K_r(j)*(C_s-Ox1(j-1)));
    M_t(j) = M(j)+M_t(j-1);
end;
sum(M)

disp('Total Amount of Oxygen Transferred Determined Successfully...')

% Determine instantaneous change in Oxygen Content for both first and second trial
Change_C1(1) = 0;
Change_C2(1) = 0;
for f = f:length(Run1Time-1);
    Change_C1(f) = Ox1(f) - Ox1(f-1);
    Change_C2(f) = Ox2(f) - Ox2(f-1);
end;

disp('Change in Measured O2 Determined Successfully...')

% dC/dt = K * C --> dC / C = K * dt --> ln C2 - ln C1 = K * t --> ln C2 = K*t1 + ln C1/ --> C2 = e^(K*t1 + ln C1)
% C2(1) = OxyCon(1);
% j = 2;
% for j = j:length(OxyTime-1);
%    C2(j) = exp(K_2(j-1)+(log(C2(j-1))));
% end;

disp('MODEL CODE SUCCESSFULLY RAN')
if Trial1 == 1;
    Time = Run1Time;
end;

if Trial2 == 1;
    Time = Run2Time;
end;

figure (2)
hold on;
stairs(Time,Cc,'b-')
hold off;
box on;

%axis([3580 5380 7.28 7.32])
    ylabel('Oxygen Concentration (mg L^{-1})')
    xlabel('Time (seconds)')
    title('Modeled Change in Oxygen Concentration Due to Power Input with No Sinks')

figure (3)
hold on;
stairs(Time,M, 'r-')
hold off;
box on;

%axis([7260 9060 11.84 14.00])
%axis([3580 5380 7.28 7.32])
    ylabel('Oxygen Transferred, (mg L^{-1})')
    xlabel('Time (seconds)')
    title('Modeled Oxygen Transfer with Time')

figure (4)
hold on;
stairs(Time,H, 'r-')
hold off;
box on;

%axis([7260 9060 11.84 14.00])
    ylabel('Depth of Water Body (cm)')
    xlabel('Time (seconds)')
    title('Modeled Water Body Depth (H) with Time')

figure (5)
hold on;
stairs(Time,M_t, 'b-')
hold off;
box on;

%axis([3580 5380 7.28 7.32])
    ylabel('Oxygen Concentration, (mg L^{-1})')
    xlabel('Time (seconds)')
    title('Modeled Water Body Dissolved Oxygen Content with Time')

if Trial1 == 1;
    figure (6)
    hold on;
stairs(Time,M_t, 'b-')
stairs(Time,Ox1, 'g-')
    hold off;
if Trial2 == 1;
    figure (6)
    hold on;
    stairs(Time,M_t, 'b-')
    stairs(Time,Ox2, 'g-')
    hold off;
    box on;
    %refline(1,0)
    %axis([3580 5380 7.28 7.32])
    ylabel('Oxygen Concentration, (mg L^{-1})')
    xlabel('Time (seconds)')
    title('Modeled Water Body Dissolved Oxygen Content with Time')
    legend('Modeled', 'Actual', 0)
end;

end;

figure (7)
hold on;
stairs(Ox2,M_t, 'k.')
stairs(Run1Time,Ox1, 'r-')
stairs(Run1Time,M_t, 'r--')
stairs(Run1Time,Ox2, 'b-')
stairs(Run1Time,M_t, 'b--')
hold off;
%refline(1,0)
%axis([6.4 8.4 6.4 8.4])
end;

figure (8)
hold on;
stairs(Ox2,M_t, 'k.')
stairs(Run1Time,Ox1, 'r-')
stairs(Run1Time,M_t, 'r--')
stairs(Run1Time,Change_C2, 'b-')
stairs(Run1Time,M, 'r--')
hold off;
%refline(1,0)
%axis([6.4 8.4 6.4 8.4])
end;
MATLAB CODE FOR WLIS ANALYSIS

close all;
clear all;

Test = 0;

Banks = 1; % Banks = 1 to run Banks Model
Trial1 = 1;

% Select the Case of interest: Case 0 is entire experiment data set.
% Case 0:  The first thing we look at (2004)
% Case 1: Viewing day 198 --> day 217

Case = 1;
% if Case ==0;
% month = 'Yearly';
% xmin = 0;
% xmax = 366;
% yminTmp = 10;
% ymaxTmp = 35;
% end;

if Case == 1;
    month = 'July & August';
    xmin = 197.5;
    xmax = 217.5;
    yminTmp = 10;
    ymaxTmp = 35;
end;

Phase_0 = 1; % NDBC Buoy MEt. Data
Phase_1 = 0; % Interpolate Bouy Data

%-------------------INTERPOLATING METEOROLOGY DATA-----------------------------

[CF_time2004 DB_ave2004] = textread ('C:\Hypoxia Research\Analysis\MATLAB Code\IRdwnAnalysis\AveDirectSolar2004_2.txt', '%f %f');

% Case selection
% CaseSWR2004_s = [1 385];
CaseSWR2004_s = [1 385];
CaseSWR2004_f = [length(CF_time2004) 841];
NsSWR_2004 = CaseSWR2004_s(Case+1);
NiSWR_2004 = CaseSWR2004_f(Case+1);
NtSWR_2004 = NiSWR_2004 - NsSWR_2004 + 1;

CFtime2004 = CF_time2004(NsSWR_2004:NiSWR_2004);

Proc_Wdir440404 = load('Proc_Wdir440404_RevA.txt');
timeWdir440404 = Proc_Wdir440404(:,1);
Proc_Wdir440404 = Proc_Wdir440404(:,2);
Wdir440404 = interp1(timeWdir440404,Proc_Wdir440404,CFtime2004);

Proc_Wspd440404 = load('Proc_Wspd440404_RevA.txt');
    timeWspd440404 = Proc_Wspd440404(:, 1);
    Proc_Wspd440404 = Proc_Wspd440404(:, 2);
    Wspd440404 = interp1(timeWspd440404,Proc_Wspd440404,CFtime2004);

    timeGst440404 = Proc_Gst440404(:, 1);
    Proc_Gst440404 = Proc_Gst440404(:, 2);
    Gst440404 = interp1(timeGst440404,Proc_Gst440404,CFtime2004);

Proc_BPR440404 = load('Proc_BPR440404_RevA.txt');
    timeBPR440404 = Proc_BPR440404(:, 1);
    Proc_BPR440404 = Proc_BPR440404(:, 2);
    BPR440404 = interp1(timeBPR440404,Proc_BPR440404,CFtime2004);

Proc_TmpC440404 = load('Proc_TmpC440404_RevA.txt');
    timeTmpC440404 = Proc_TmpC440404(:, 1);
    Proc_TmpC440404 = Proc_TmpC440404(:, 2);
    TmpC440404 = interp1(timeTmpC440404,Proc_TmpC440404,CFtime2004);

Proc_WTmpC440404 = load('Proc_WTmpC440404_RevA.txt');
    timeWTmpC440404 = Proc_WTmpC440404(:, 1);
    Proc_WTmpC440404 = Proc_WTmpC440404(:, 2);
    WTmpC440404 = interp1(timeWTmpC440404,Proc_WTmpC440404,CFtime2004);

Proc_DwpC440404 = load('Proc_DwpC440404_RevA.txt');
    timeDwpC440404 = Proc_DwpC440404(:, 1);
    Proc_DwpC440404 = Proc_DwpC440404(:, 2);
    DwpC440404 = interp1(timeDwpC440404,Proc_DwpC440404,CFtime2004);

%******************************************************************************************
% LOAD FRG TIME AND RAIN RATE DATA

[load_WxB_FRG_Year load_WxB_FRG_Month load_WxB_FRG_Day load_WxB_FRG_Hour load_WxB_FRG_RainRate] = textread('C:\Senior Research\Analysis\Proc_KFRG-2004.txt', '%f %f %f %f %f');

% Case selection
%      0  1  2  3  4  5
CaseFRG2004_s = [1  4582];
CaseFRG2004_f = [length(load_WxB_FRG_Year)  5020];
NsFRG_2004 = CaseFRG2004_s(Case+1);
NfFRG_2004 = CaseFRG2004_f(Case+1);
NtFRG_2004 = NfFRG_2004 - NsFRG_2004 + 1;

FRG_RainData = load('Proc_KFRG-2004.txt');
WxB_FRG_Year = load_WxB_FRG_Year(NsFRG_2004:NfFRG_2004);
WxB_FRG_Month = load_WxB_FRG_Month(NsFRG_2004:NfFRG_2004);
WxB_FRG_Day = load_WxB_FRG_Day(NsFRG_2004:NfFRG_2004);
WxB_FRG_Hour = load_WxB_FRG_Hour(NsFRG_2004:NfFRG_2004);
WxB_FRG_RainRate = load_WxB_FRG_RainRate(NsFRG_2004:NfFRG_2004);

% Calculating TIME
WxB_FRG_DOY = datenum(WxB_FRG_Year,WxB_FRG_Month,WxB_FRG_Day)-WxB_FRG_Year*365.25+15;
WxB_FRG_Time = WxB_FRG_DOY+WxB_FRG_Hour/24;
WxB_FRG_JTime = WxB_FRG_Time

FRG_RR = interp1(WxB_FRG_JTime,WxB_FRG_RainRate,CFtime2004);

% LOAD HPN TIME AND RAIN RATE DATA

[load_WxB_HPN_Year load_WxB_HPN_Month load_WxB_HPN_Day load_WxB_HPN_Hour
load_WxB_HPN_RainRate] = textread ('C:\Senior Research\Analysis\Proc_KHPN-2004.txt', '%f %f %f %f');

% Case selection
% 0 1 2 3 4 5
CaseHPN2004_s = [1 4581];
CaseHPN2004_f = [length(load_WxB_HPN_Year) 5017];
NsHPN_2004 = CaseHPN2004_s(Case+1);
NhHPN_2004 = CaseHPN2004_f(Case+1);
NhHPN_2004 = NhHPN_2004 - NsHPN_2004 + 1;

% HPN_RainData = load('Proc_KHPN-2004.txt');
WxB_HPN_Year = load_WxB_HPN_Year(NsHPN_2004:NfHPN_2004);
WxB_HPN_Month = load_WxB_HPN_Month(NsHPN_2004:NfHPN_2004);
WxB_HPN_Day = load_WxB_HPN_Day(NsHPN_2004:NfHPN_2004);
WxB_HPN_Hour = load_WxB_HPN_Hour(NsHPN_2004:NfHPN_2004);
WxB_HPN_RainRate = load_WxB_HPN_RainRate(NsHPN_2004:NfHPN_2004);

% Calculating TIME
WxB_HPN_DOY = datenum(WxB_HPN_Year,WxB_HPN_Month,WxB_HPN_Day)-WxB_HPN_Year*365.25+15;
WxB_HPN_Time = WxB_HPN_DOY+WxB_HPN_Hour/24;
WxB_HPN_JTime = WxB_HPN_Time

HPN_RR = interp1(WxB_HPN_JTime,WxB_HPN_RainRate,CFtime2004);

% LOAD LGA TIME AND RAIN RATE DATA

[load_WxB_LGA_Year load_WxB_LGA_Month load_WxB_LGA_Day load_WxB_LGA_Hour
load_WxB_LGA_RainRate] = textread ('C:\Senior Research\Analysis\Proc_KLGA-2004.txt', '%f %f %f %f');

% Case selection
% 0 1 2 3 4 5
CaseLGA2004_s = [1 4679];
CaseLGA2004_f = [length(load_WxB_LGA_Year) 5119];
NsLGA_2004 = CaseLGA2004_s(Case+1);
NfLGA_2004 = CaseLGA2004_f(Case+1);
NfLGA_2004 = NfLGA_2004 - NsLGA_2004 + 1;

% LGA_RainData = load('Proc_KLGA-2004.txt');
WxB_LGA_Year = load_WxB_LGA_Year(NsLGA_2004:NfLGA_2004);
WxB_LGA_Month = load_WxB_LGA_Month(NsLGA_2004:NfLGA_2004);
WxB_LGA_Day = load_WxB_LGA_Day(NsLGA_2004:NfLGA_2004);
WxB_LGA_Hour = load_WxB_LGA_Hour(NsLGA_2004:NfLGA_2004);
WxB_LGA_RainRate = load_WxB_LGA_RainRate(NsLGA_2004:NfLGA_2004);

% Calculating TIME
WxB_LGA_DOY = datenum(WxB_LGA_Year,WxB_LGA_Month,WxB_LGA_Day)-WxB_LGA_Year*365.25+15;
WxB_LGA_Time = WxB_LGA_DOY+WxB_LGA_Hour/24;
WxB_LGA_JTime = WxB_LGA_Time
LGA_RR = interp1(WxB_LGA_JTime,WxB_LGA_RainRate,CFtime2004);

figure(101)
hold on;
stairs(CFtime2004,FRG_RR, 'r-')
stairs(CFtime2004,HPN_RR, 'b-')
stairs(CFtime2004,LGA_RR, 'g-')
hold off;

box on;
ylabel('Rain Rate, \ (in \ hr^{-1})')
xlabel('Julian Day')
title('Average Rain Rate, r, for WLIS')

if Banks == 1;
if Test == 1;
   C_o = 7.3; % Oxygen Concentration just prior to rain's addition
   C_r = 9.21; % mg/l - pressure 765 torr @ 19.5 C
   U = 11.1; % Wind Magnitude (m/s)
   r = 15; % mm/hr experimental with water density @ 20 C
   K_star = 0.0246; % b1/b2 (cm/s)
   z = 2000; % Altitude above MSL in meters
   C_s = C_r; % Oxygen Saturation level of Water Body
   H_o = 0; % initial Depth of Water Body
end;

if Trial1 == 1;

%----------------Determining DO content of Rain Fall---------------------

% Determining Lifted Condensation Level

a = 0.125; % Km/C
z_LCL = a * (TmpC440404-DwpC440404); % Lifted Condensation Level

Dry_LR = 9.8; % Dry Lapse rate (C/Km)
T_LCL = TmpC440404-Dry_LR * z_LCL;

T_w = T_LCL + Dry_LR * z_LCL; % Wet-bulb Temperature

% Determining Vapor Pressure

p = 1;
for p = p:length(CFtime2004);
   P_vap(p) = 0.611 * exp(5423 * ((1/273.15)-(1/(DwpC440404(p)+273.15))));
end:
% Determining Dissolved Oxygen Content of Rain Fall

c = 1;

for c = c:length(CFtime2004);
    C_r(c) = ((((BPR440404(c)*0.75) - P_vap(c))).*(0.678))./(35+T_w(c)); %Note: Equation only applicable from 0 C to 30 C
    C_s = (((BPR440404(c)*0.75) - P_vap(c)).*(0.678))./(35+WTmpC440404(c)); %Note: Equation only applicable from 0 C to 30 C
end;

%----------------Setting Wind Speed---------------------------------------------
U = Wspd440404;  % Wind Magnitude (m/s)

%----------------Determining Rain Rate at Bouy---------------------------------

r_in = (FRG_RR + HPN_RR + LGA_RR)/3;% in.hr
r = r_in * 25.4 ; %mm/hr

%----------------Determining Oxygen Saturation Level of Water Body

C_o = 7.3; % Oxygen Concentration just prior to rain's addition
K_star = 0.0246; % b1/b2 (cm/s)
z = 0; % Altitude above MSL in meters
K_s = C_r; % Oxygen Saturation level of Water Body
H(1) = 1000; % initial Depth of Water Body

i=1;

for i = i:length(CFtime2004);
    K_o(i) = (10^(-6))*((8.43*(U(i)^(1/2)))-(3.67*U(i))+(0.43*(U(i)^2)))*100; % Determining Oxygen Transfer Coefficient due to only wind
    P_t(i) = (2.39 + (0.103*(10^(-3))*z))*(r(i)^(1.26)); % Determining the Total Power input due to rain
    K_r(i) = (0.0283 * (10^-3)) .* P_t(i); % Determining the Oxygen Transfer Coefficient due to rain
    K_t_NT(i) = (K_o(i)) - ((K_o(i).*K_r(i))./(K_star)) + K_r(i); % Determining the total oxygen transfer coefficient
end;

K_t = K_t_NT'; % Transposing K_t Matrix for comparison with real data.

disp('K_t Determined Successfully...')

figure (2)
hold on;
stairs(CFtime2004,K_o, 'r-')
hold off;
box on;
axis([197.5 217.5 -0.00025 0.002 ])
refline(0,0)
ylabel('Surface Reaeration Coefficient of Wind, $K_{0}$ (cm s^{-1}))'\nxlabel('Julia Day')\ntitle('Surface Reaeration Coefficient of wind, $K_{0}$, with Time')

figure(3)\nhold on;\nstairs(CFtime2004,P_t, 'r-')\nhold off;\nbox on;\n\nylabel('Power Input from Rain, $P_{T}$ (dyne cm^{-2} s^{-1}))'\nxlabel('Julia Day')\ntitle('Power Input from Rain, $P_{T}$, with respect to Time')

figure(4)\nhold on;\nstairs(CFtime2004,K_r, 'r-')\nhold off;\nbox on;\n\nylabel('Surface Reaeration Coefficient of Rain Induced Turbulence, $K_{r}$ (cm s^{-1}))'\nxlabel('Julia Day')\ntitle('Surface Reaeration Coefficient of Rain Induced Turbulence with Time')

figure(5)\nhold on;\nstairs(CFtime2004,K_t, 'r-')\nhold off;\nbox on;\n\nylabel('Total Surface Reaeration Coefficient, $K_{2}$, (cm s^{-1}))'\nxlabel('Julian Day')\ntitle('Total Surface Reaeration Coefficient, $K_{2}$, with Time')

%-----------------------Total Amount of Oxygen Transferred\nj=1;\nfor j = j:length(CFtime2004);\n  M(j) = (((r(j) * C_r(j))/36000);\nend;\nsum(M)\ndisp('Total Amount of Oxygen Transferred Determined Successfully...')

disp('MODEL CODE SUCCESSFULLY RAN')

figure (6)\nhold on;\nstairs(CFtime2004,M, 'r-')\nhold off;\nbox on;\n%axis([3580 5380 7.28 7.32])\n\nylabel('M, (cm mg L^{-1} s^{-1}))'\nxlabel('Julian day')\ntitle('M vs. Time')

end;\nend;
end;